

The Development of a Technological Processor as a Part of a Workpiece Programming System

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The cost of turning on NC-lathes is substantially sensitive to cutting conditions. The use of optimum cutting conditions is limited by a large number of constraining factors such as chip disposal, chucking possibility, available power, required accuracy etc. The present way of work preparation, even when using the available workpiece programming systems, does hardly allow of generating acceptable values for the cutting variables. Moreover, the combination of geometrical and a technological processor in one workpiece programming system will save substantial time in work preparation.

Starting from a former reported development, this article deals with the development of a technological processor of improved design and extended possibilities, as for instance the handling of the chucking problem, as a part of a complete workpiece programming system. At choice the output of this processor may be presented in graphes, showing the limited working area and indicating the preferential working point.

Another feature is the possibility of automatic tool selection by comparing the working area of the machine tool with the working areas of the different potential tools.

The overall system design is modular and well structured to further portability and flexibility.

1. INTRODUCTION

As the machining process on lathes becomes more automated, the ratio of actual cutting time to total machining time increases and hence the influence of cutting conditions on the economics of the machining operation becomes increasingly important. Due to the higher investments involved in NC-machining, compared with conventional operations, the sensitivity of the machining costs to deviations of the cutting conditions from their optimum value is much more significant. In this context by optimum is meant: those values of feed, speed and depth of cut that are suited for machining in the most economical way.

The economical objective can be maximum production rate, minimum production costs, maximum profit rate or any other required criteria.

In work preparation for NC-machines there is an increasing use of workpiece programming systems. Most of the available workpiece programming systems only deal with geometrical problems, so as to generate the tool path, while the machining conditions e.g. feed and speed have to be generated in the conventional way. Other systems may provide machining conditions but in general, economic acceptable values specified within the constraints of a given lathe are not calculated.

An increase of the automation level in a production system causes a decrease in flexibility. In this sense flexibility is defined as the possibility to accept and produce a number of different small batches of products, each of them with their own specific difficulties regarding geometry and material properties, within a range of acceptable costs. For small batches the cost of work preparation greatly affects the machining cost per product. Only the use of a workpiece programming system which is capable of generating economic cutting conditions, appropriate tooling and effective machining methods and sequences within a limited period of time allows profitable production of small batches on NC-machines.

The technological processor, designed to generate the best economic cutting conditions forms a part of an overall workpiece programming system called ROUND.

2. THE WORKPIECE PROGRAMMING SYSTEM ROUND.

Round is a workpiece programming system for machining operations on lathes. The system consists of a number of different modules for respectively input, classification of geometrical configurations and determination of machining methods, tool-selection, the selection of economic cutting conditions, toolpath generation and output.

Every module in fact represents a separate program which is loaded by a control module. A background memory is used for the transfer of data from one module to another. At the end of its execution the module will start the control module, which in turn will start the next module. It is not necessary for the user to interfere but a message will be given every time when a module is stopped or started. The execution of the program can be stopped or suspended after each specified module.

This modular system design is chosen to enable implementation on minicomputers and to simplify the maintenance of the system. The design philosophy used for Round is the same as previously reported of in the case CUBIC, a workpiece programming system for machining centres [9]

Another feature of the system design is represented by the so called 'fixed files'. All workpiece-independent data are stored here. The fixed files include data about machine tools, materials, tools and manufacturing procedures. It is possible for each user to adapt those files to his own needs by changing their contents with an off-line utility program.

Every module consists of two parts: a service part that contains subroutines which perform the communication between the module and the files or other modules, and a problem-part that contains the routines which handle the problem the module was designed for. The function of the different modules are briefly explained below.

2.1 THE INPUT MODULE.

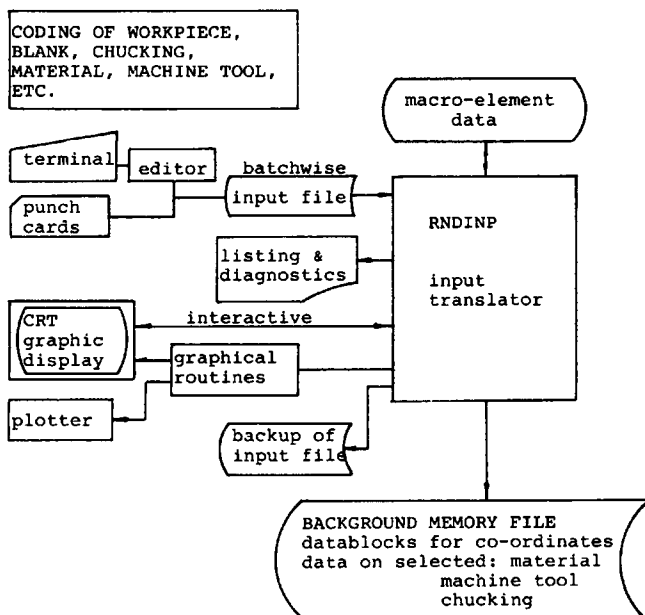


Fig. 1 Blockdiagram of the input module.

Fig. 1 shows the functions of the input module. The input can be realized either interactively or batchwise. The input data can be checked with the aid of graphic display of the blank, the workpiece and the chucking. The definition of both blank and part geometry is given by making up shape compositions of basic elements like cylinders, tapers, planes, arcs etc.

The user can compose so called 'macro-elements' which can be stored to be recalled when necessary.

2.2 THE CLASSIFICATION MODULE.

This module contains procedures to recognize different typical shape compositions which have to be machined by a predetermined method and a given machining sequence.

Such a method may contain for example a packed drilling cycle followed by boring cycles to machine

the inside of the workpiece or procedures about longitudinal turning, facing or contouring a given shape. The module receives data from the method file and the materials data file.

2.3 THE TOOL SELECTION MODULE.

For each machining operation the module selects the appropriate toolholder with respect to the geometry of the shape composition to be made. The grade of the carbide insert is selected on the grounds of the properties of the workpiece material and the type of machining operation e.g. roughing or finishing, threading or drilling etc. The module receives data from the toolholder data file and the materials data file.

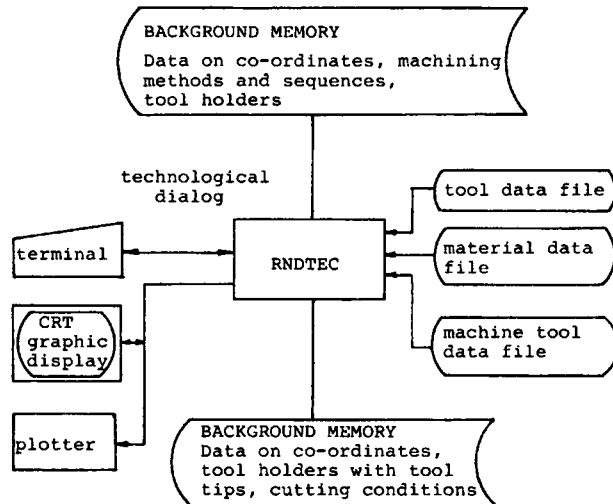


Fig. 2 Blockdiagram of the technological processor.

2.4 THE TECHNOLOGICAL PROCESSOR.

The technological processor (Fig. 2) calculates the most economic cutting conditions. A dialog with the user about alterations in the suggested setup of the machining operations is possible. This is also the way in which, for example, a different choice of carbide insert has to be brought in when the program does not arrive at an appropriate solution. A graphical presentation of the results of the optimization can be given. The calculation of the cutting conditions will be discussed in the next section.

2.5 THE SORTING AND TOOL CO-ORDINATE MODULE.

This module reorganizes the different cutting operations in such a way that equivalent operations will be performed with the same tool and in a proper sequence. If wanted, the cutting conditions can slightly be adapted to ensure that an integer number of products are machined within the tool life of the different tools. The module fills the tool co-ordinate file consisting of different tool records each with the matching co-ordinate records.

2.6 THE POSTPROCESSOR.

The tool co-ordinate file is adapted by this module into a form that is accepted by the machine tool involved. Every type of machine tool requires a specific postprocessor. An external utility program to generate post-processors is being developed.

3. THE STATE OF DEVELOPMENT.

The state of development of ROUND is that all the modules mentioned have been designed. The input-module is ready for use, so are the utility programs to create and edit the fixed files. Both the method file and the classification module are under development. The description of the technological processor of which the problem routine part has been implemented, follows next.

4. THE DESCRIPTION OF THE TECHNOLOGICAL PROCESSOR.

After the machining methods and the subsequent machining sequences have been determined and the appropriate tools for all machining operations have been selected, the most economic cutting conditions can be determined. This can be done by calculating the intersection of the spatial representation of the objective cost function with the limited area of solutions for the cutting conditions which can be reached technically. The limited area is primarily determined by the specifications of the tool and the machine tool. The geometrical and accuracy specifications of the workpiece are also a major factor in the determination of the ultimate available optimization area.

The total machining cost yield from:

$$C = c_m \cdot t_a + c_m \sum_{j=1}^m \frac{v_j d_j l_j}{s_j v_j} + (c_m t_s + C_0 s) \sum_{j=1}^m \frac{v_j d_j l_j}{s_j v_j T_j} + c_m \sum_{j=1}^m t_{p_j} \quad (1)$$

$$\text{with } T_j = \left[\frac{K \cdot K' \cdot v_o^m}{v \cdot h e_j^i} \right]^{1/n} \quad (2)$$

The derivation of the equations (1) and (2) are given in [2].

The objective is minimum production cost for $C_0=1$. For $C_0=0$ the cost of the tools are neglected and the objective is maximum production rate. For $0 < C_0 < 1$ yields a compromise solution to obtain a.o. maximum profit rate.

In Fig. 3 an indication is given about both the shape of the surfaces of equal costs in the v - a - s space and the area of the technical possible combinations of feed, speed and depth of cut [7]. First, the limits of the area for v , a and s are calculated. These limits represent the restrictions set by the machine tool, the chucking and the cutting tool, together with the constraints set by the geometrical accuracy, the surface quality of the product and the behaviour of the cutting process, the latter regarding dynamic stability and chip disposal.

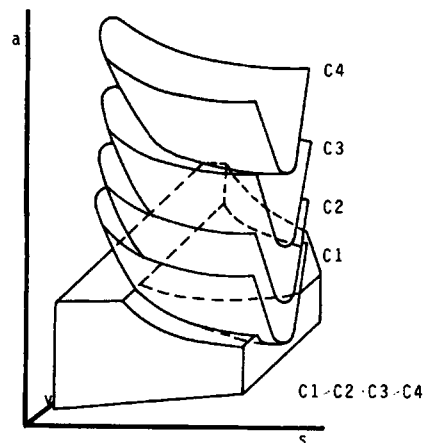


Fig. 3 The shape of the surfaces of equal costs and the area of technically possible combinations of v , a and s .

4.1 THE CONSTRAINTS.

The constraints of the different variables of the process are denoted in Table 1

Most of the constraints are quite obvious, such as

- the maximum machinable diameter (DMMAX)
- the feed and speed range of the machinetool (SMMAX, SMMIN, NMMAX, NMMIN)
- the feed and speed range of the tool (STMIX, STMIN, VTMIX, VTMIN)
- the maximum cutting depth of the tool (ATMAX)
- the maximum allowable tool load (FTMAX) and
- the maximum depth of cut, restricted by the geometrical differences between the blank and the workpiece (AWMAX)

These constraints need not to be discussed further. Some others have been described in a preceding paper [1] and are still calculated in the same way.

These are:- the maximum feed controlling surface roughness (SWMAX) and
- the maximum depth of cut controlling dynamic stability of the process (APMAX).

	Machine tool	Tool	Work piece	Process/Chucking
Depth of cut		ATMAX ATMIN	AWMAX	APMAX
Feed	SMMAX SMMIN	STMAX STMIN	SWMAX	
Area of chip				ASMAX
Slenderness ratio				DLMAX
Cutting speed		VTMAX VTMIN		
Number of revolutions	NMMAX NMMIN			NCMAX
Cutting force		FTMAX	FWMAX	
Torque	MMMAX			MCMAX
Power	PMMAX			
Chucking Force			FCMAX	
Max. turning diameter	DMMAX			

Table 1. Survey of constraining factors.

The calculation of the maximum slenderness ratio of the cut (DLMAX) and the maximum area of the chip (ASMAX), both influencing chip breakage and consequently controlling chip removal, is carried out by a procedure using formulae derived from [2].

The spindle torque of the machine tool is calculated from

$$MM = \frac{PMMAX}{2 \cdot \pi \cdot n_i} \quad \text{for } n > n_i \quad (3)$$

and $MM = MMAX$ for $n < n_i$

The spindle power yield from

$$PM = PMMAX \quad \text{for } n \geq n_i$$

$$\text{and } PM = MMAX \cdot 2 \cdot \pi \cdot n \quad \text{for } n < n_i \quad (4)$$

For machine tools with DC-drives and a given gear ratio, the number of revolutions n is determined by the transition point of the range of constant spindle torque and the range of constant spindle power (Fig. 4).

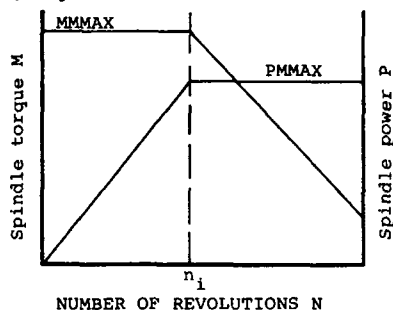


Fig 4. Spindle power and torque as a function of the number of revolutions for a Lathe with DC-drive.

The calculation of the constraints imposed by the chucking method is added to the program. The rather extensive calculation is absolutely necessary since in many cases the chucking is a major constraining factor. The roundness of relatively thinwalled clamped workpieces is strongly influenced by the radial chucking force (FCMAX) which in turn sets the constraints for the maximum allowable torque. Depending on the centripetal force, the maximum torque that can be transferred by the chuck (MCMAX) is related to the rotational speed. The chucking is considered to be safe for rotational speeds below NCMAX, the latter giving the number of revolutions for which 33% of the chucking force at $n=0$ (FCHO) is left. Some of the equations which are used will be explained in Appendix 1.

A set of formulae used to calculate the maximum cutting force imposed by cylindricity-tolerances of the workpiece (FWMAX) will be discussed in Appendix 2. All constraints can be expressed in functions of v , a and s .

4.2 THE OPTIMIZATION STRATEGY.

First, the technical solution space, expressed in terms of v , a and s , is composed by the determination of the most constraining factors.

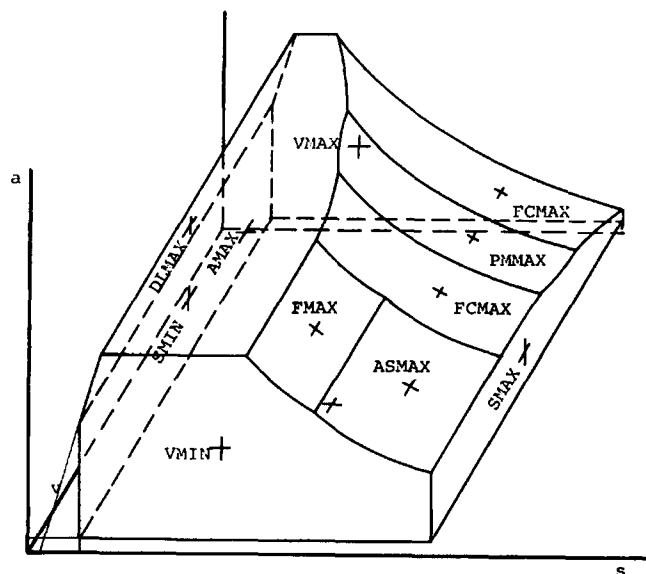


Fig. 5 The Space of Solutions for v , a and s

Fig (5) gives an indication of a possible solution space.

Subsequently the equations of the different limiting surfaces are substituted into the cost equation in such a way that the variable s is eliminated. Putting the partial differential of the resulting equations to zero e.g. $\partial C / \partial a = 0$ (5), gives a number of local optima for the depth of cut (a_{opt}). It occurs that for realistic values of the tool life exponents i and n , only an optimum can be found on the a_{max} plane. So the procedure to calculate the cutting conditions is

step 1: Calculate the maximum depth of cut.

The remainder of the optimization is reduced to a two dimensional problem on the a_{max} plane.

step 2: Calculate the maximum feed for the given depth of cut.

step 3: Calculate the optimum speed corresponding to a_{max} and s_{max} .

4.3 GRAPHIC PRESENTATION OF THE RESULTS.

It is difficult to visualize the results of the optimization in a three dimensional diagram, because the exact shape of the surfaces at every point cannot be drawn. For this reason the results are presented in a set of two dimensional diagrams.

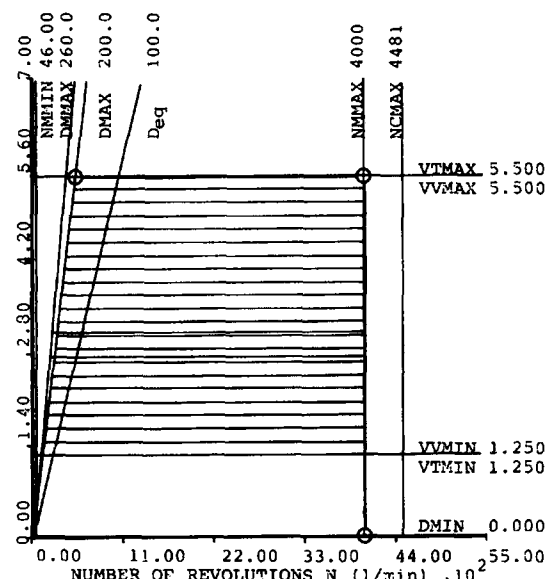


Fig. 6 Number of Revolutions vs. Cutting Speed (Finishing)

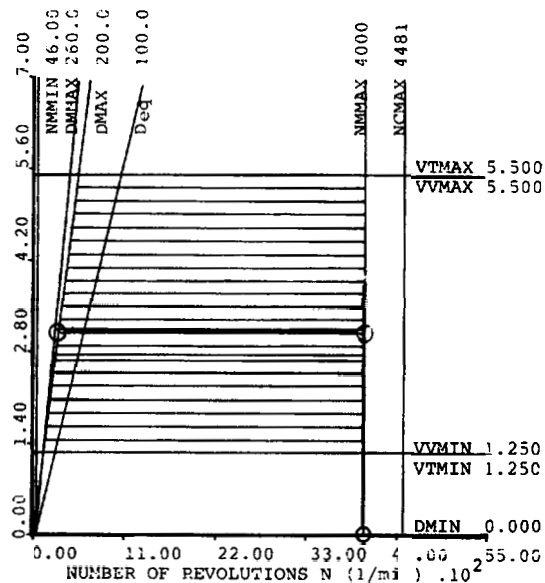


Fig. 7 Number of Revolutions vs. Cutting Speed (Roughing)

The calculation of economic cutting conditions for a facing operation will be explained by means of these diagrams. The figures 6, 8 and 10 represent the finishing cut, while the figures 7, 9 and 11 concern rough cutting. Figs. 6 and 7 show the relation between the cutting speed and the number of revolutions of the machine tool and are particularly helpful in evaluating facing and tapering operations. The constraints for maximum and minimum cutting speed are calculated reckoning with both the diameter range of the cut and the RPM-range of the machine tool. For turning operations the shaded areas in Figs. 6 and 7 become lines. The equivalent diameter for facing and tapering is given by

$$D_{eq} = \left(\frac{D_{max}^{(1+1/n)} - D_{min}^{(1+1/n)}}{(1+1/n)(D_{max} - D_{min})} \right)^n \quad (6) \text{ for constant RPM} \quad [3]$$

$$D_{eq} = \frac{D_{max} + D_{min}}{2} \quad (7) \text{ for constant cutting speed}$$

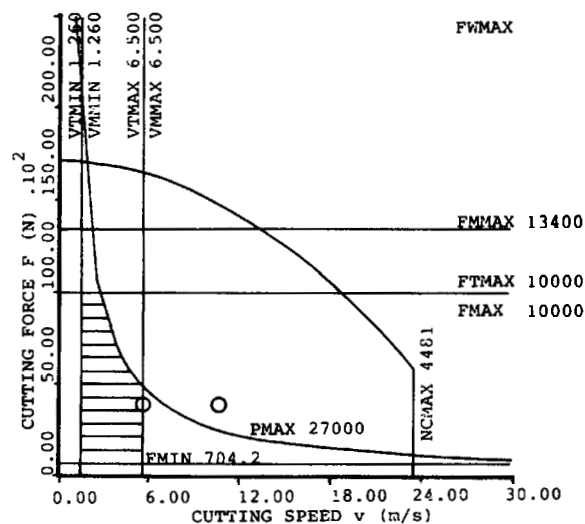


Fig. 8 Cutting Force vs. Cutting Speed (Finishing)

A turning operation on the equivalent diameter D over an equal cutting length will result in an equal amount of toolwear. A facing operation up to the center of the workpiece with constant cutting speed will be carried out with constant RPM until NMMAX is reached. In that case cutting speeds below VTMIN have to be allowed for small diameters. The cutting force as a function of the cutting speed is represented in Figs. 8 and 9.

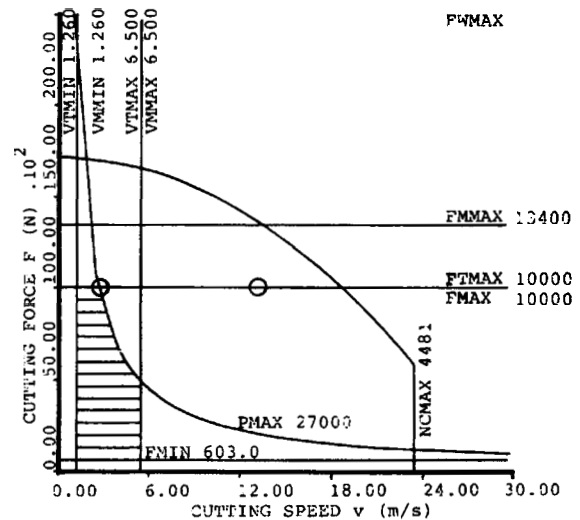


Fig. 9 Cutting Force vs. Cutting Speed (Roughing)

The limits of the speedrange (see Figs. 6 and 7) are shown in these figures. The optimum cutting speed is to be found within these limits. The cutting force constraint called FMAX is the minimum of the force values calculated to satisfy the other relevant constraints i.e. the tolerances of the workpiece (FWMAX), the maximum spindle torque (FMMAX = MMAX . 2 / D), the load on the tool (FTMAX) and the chucking torque (CFMAX = MCNAX . 2 / D).

Within the shaded area technically possible values of v_{opt} are obtained. The diagrams giving the cutting speed vs. depth of cut (Figs. 10 and 11) show the areas of solutions for a and s limited by the different constraints.

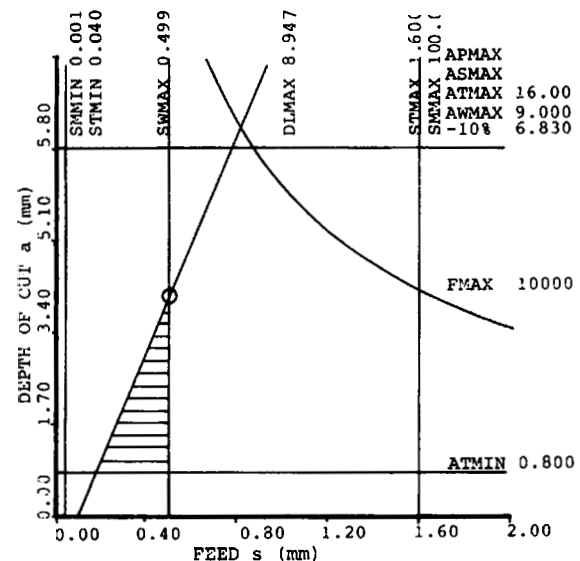


Fig. 10 Feed vs. Depth of Cut (Finishing)

The model that is used to calculate the cutting force mentions the cutting speed [1] which is not known in the initial calculation. To calculate feed and depth of cut an iterative procedure is used. A lot of attention is paid to the development of algorithms to obtain reasonable starting values. When later on the actual cutting force happens to be higher than predicted, the only way to reduce it is by decreasing the feed.

To create a small range in which the feed can be reduced without getting into trouble by crossing the constraints for the slenderness ratio and the minimum feed, the maximum depth of cut is reduced by a certain percentage allowing a small readjustment of the feed.

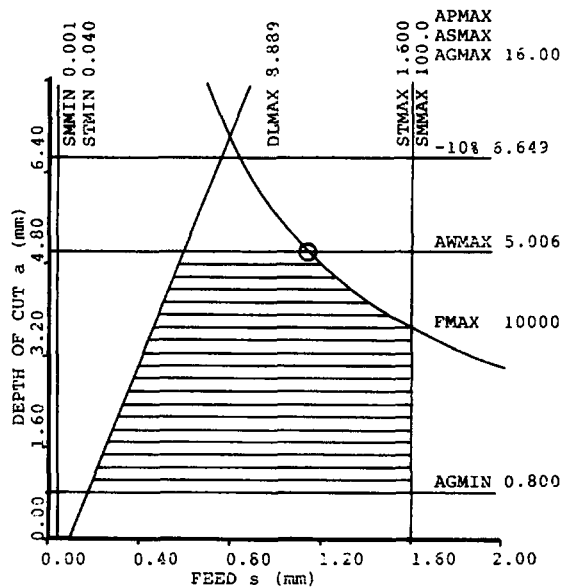


Fig. 11 Feed vs. Depth of Cut (Roughing)

When for a given machining operation equal depths of cut are chosen, only a limited number of a -values can be obtained due to the fact that an integer number of cuts have to be made. The highest possible a -value will be taken and the value of s is calculated. Subsequently the optimum value for the cutting speed marked in Figs. 8 and 9 will be computed. A list of data giving the calculated cutting conditions, costs, times and toolwear can be produced.

The list also contains additional data regarding spindle power, hydraulic pressure for chucking and the theoretical tolerances on roundness, cylindricity and surface roughness of the workpiece. The list concludes with the total machining cost, the total machining time per product and the number of products to be machined per cutting edge. The graphical output can be used to evaluate suspicious values of the cutting conditions and to find out which the dominating constraints are. The graphs can also be very useful in checking the coincidence of the working areas of the machine tool, the chuck and the tools involved. A better choice of tool or machine tool may result from this. During the technological dialog, all kinds of alterations in technological data can be made. Examples are the changing of the chucking, the geometry of the tool tip and the carbide grade.

5. CONCLUDING REMARKS.

The technological processor described, forms the pivot of the workpiece programming system ROUND. Its purpose is to enable profitable production of small batches on NC-lathes. It gives reliable values for machining times and machining costs (depending on the specifications of the lathe, the machine costs per hour and the tool costs per cutting edge) which have to be offered to the planning department as the basis for the correct planning of the production system. Procedures for automatic adaption of data based on information put in either manually or given by sensors, (the so called "heuristic feedback") will be an essential part of future research. The program can be used independently of the ROUND-system as a powerful tool for evaluating the sensitivity of machining costs and machining times to the different process variables as a result of changes in the specifications of the tool, the machine tool or the tolerances of the workpiece. In this way the program can also be used as a valuable design tool in order to obtain a balanced machine tool design.

6. ACKNOWLEDGEMENT.

The author wishes to acknowledge the vital contributions to the overall system design by H. Stoltenkamp.

APPENDIX 1

Calculations on the chucking.

For a hydraulically operated chuck of the wedge type, the reduced radial force of a jaw acting on a product at $n = 0$ can be derived from $F_{CH}/n_j = K_1 \cdot p \cdot R_2 \cdot R_3$ with K_1 = area of the hydraulic cylinder per jaw, p = hydraulic pressure.

$$R_2 = \frac{1 - \mu \tan \alpha}{\tan \alpha + \mu} \quad \text{and} \quad R_3 = \frac{1}{1 + 2\mu(L/l)} \quad [8]$$

where α = the inclination angle of the wedge
 μ = the friction coefficient (for the wedge type of chuck $\mu \approx 0.05$)
 L = the distance between the guideways of the jaw and the point of application of the radial chucking force
 l = the length of the guideways of the jaws

The sensitivity of this force to the ratio L/l is rather small ($1 < L/l < 2$) and hence a good approximation is made by putting L as the length of the jaw. The equation can be written as $F_{CH}/n_j = C_{CH} \cdot p$. The value of C_{CH} can be determined by experiments.

The maximum of the radial chucking force at $n = 0$ is set by the tolerances for the roundness of the workpiece such that

$$F_{CH}^{MAX} = \frac{R_n \cdot E \cdot L_{CH}}{\lambda^3 \cdot K_n \cdot K_B}$$

where R_n : tolerance for roundness, L_{CH} : chucking length, $\lambda = (D_{CH} + D_1)/2(D_{CH} - D_1)$

$$K_B = (0.505 + 0.495 \cos(4\pi_j B/D_{CH}))^{0.6} \quad [2]$$

$$K_n = \frac{6}{n_j} \left(\frac{1}{\sin(\pi/n_j)} - \cot \frac{\pi}{n_j} - \frac{\pi}{2n_j} \right) \quad [8]$$

B = the width of the jaw

The reduction of the chucking force due to the centripetal force is given by

$$\Delta F_{CH} = (M_R - M_1 R_1) \omega^2 / C_1$$

where

$$C_1 = \frac{1 + \frac{R_r}{R_k} (h_k - h_F) h_k + \frac{R_r}{R_w}}{1 + \frac{R_r}{R_k} (h_k - h_F) h_{ks}} \quad [5]$$

C_1 introduces the influence of the tilting stiffness of the jaw

M : mass of the jaws

M_1 : mass of the compensation weights

R : distance of the center of gravity of the jaws to the centerline

R_1 : distance of the center of gravity of the counterweights to the centerline

R_r : radial stiffness of the jaws

R_k : tilting stiffness of the jaws

R_w : radial stiffness of the workpiece

h_k : lever of the chucking force

h_F : distance of the tilting axis of the jaw to its guideways

h_{ks} : lever of the centripetal force

The effective chucking force is then $F_{eff} = F_{CH} - \Delta F$ where F_{CH} is the chucking force at $n = 0$

ΔF is the decrease of the chucking force due to centripetal force

The chucking torque is given by

$$M_{CH}^{MAX} = \frac{2F_{eff}}{D_{max}} \left\{ \frac{1}{3\mu D_{CH}} + \frac{1}{y} L^2 + \frac{1}{2} \left(\frac{D_{max}}{5} \right)^2 \right\} \quad [5]$$

$$\text{with } y = 3(2L_{CH} + (1 + \sqrt{3}/4)\mu D_{CH}) \quad [2]$$

$$\mu = 0.15 \text{ for soft jaws} \quad [8]$$

$$\mu = 0.35 \text{ to } 0.8 \text{ for hard jaws}$$

The maximum number of revolution of the chuck is reached when 33% of the chucking force at $n = 0$ is left, i.e. $\Delta F = 0.66 F_{CH}^{MAX}$, so

$$N_{CH}^{MAX} = \sqrt{\frac{F_{CH}^{MAX} \cdot C_1}{3\pi(M \cdot R - M_1 R_1)}}$$

APPENDIX 2

Computation of the deflection of the workpiece due to the cutting force.

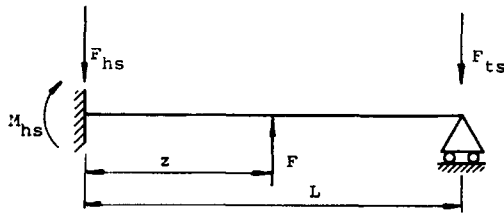


Fig. 12 Force balance of the Workpiece

For a cylindrical massive or hollow workpiece, the deflection of the workpiece at a distance z from the headstock yield from $X(z) = F \cdot U_2$

$$U_2 = \frac{z - U_1 L}{X_M} + \frac{1 - U_1}{X_F} + \frac{z - U_1 L}{\theta_M} + \frac{1 - U_1}{\theta_F} + \frac{z^3}{3EI} + \frac{U_1 z^3}{6EI} - \frac{U_1 L}{2EI} \quad [2]$$

$$U_1 = \frac{\frac{z}{X_M} + \frac{1}{X_F} + \frac{zL}{\theta_M} + \frac{L}{\theta_F} + \frac{3z^2 L}{2EI} - \frac{z^3}{2EI}}{\frac{L^3}{EI} + \frac{1}{X_C} + \frac{L}{X_M} + \frac{1}{X_F} + \frac{L}{\theta_M} + \frac{L}{\theta_F}} \quad [2]$$

where X_M , X_F , θ_M and θ_F are the values of the stiffness matrix of the workpiece. The radial stiffness of the tailstock is given by X_C . When the tailstock is not used $X_C = 0$ and hence $U_1 = 0$. The absolute and relative deflections per unit of force are calculated with this equation. The relative deflection shown in Fig. 13 must not exceed the cylindricity tolerance of the workpiece and the absolute deflection multiplied by two must not exceed the diameter tolerance.

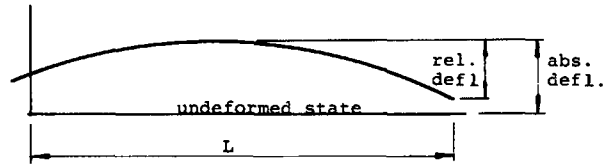


Fig. 13 Relative and Absolute Deflections

To calculate the deflection of geometrically complex workpieces the following method is used. First, the deflection at L_1 (distance of the segment to be machined to the chuck) is calculated (fig. 14). Subsequently the deflections in a number of points with a spacing of $D/4$ are computed. The difference between the maximum and minimum value of the deflection determines the cylindricity of the segment. The sum of the maximum positive deflection and the deflection at L_1 determines the diameter accuracy of the segment.

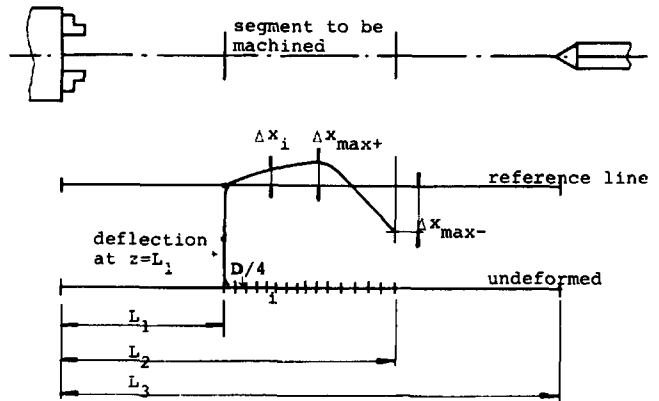


Fig. 14 Calculation of deflection for complex geometries.

NOMENCLATURE

a	[mm]	depth of cut
B	[mm]	width of the jaw
C	[Dfl]	machining costs per product
Co	[-]	factor determining optimization crit.
C1	[-]	reduction factor for the tilting moment
CCH	[m ²]	chucking constant
cm	[Dfl/s]	cost of labour and machine tool per sec.
cs	[Dfl]	tool cost per edge
Deq	[mm]	equivalent diameter
DCH	[mm]	chucking diameter
Di	[mm]	inside diameter of the workpiece
Dmax	[mm]	maximum diameter of cut
Dmin	[mm]	minimum diameter of cut
d	[mm]	diameter
E	[N/m ²]	Youngs modulus
FCH	[N]	radial chucking force
Feff	[N]	effective radial chucking force
Fv, Fp	[N]	cutting force, thrust force
he	[mm]	equivalent chip thickness
I	[m ⁴]	second moment of area
i, m, n		exponents in tool life equation
K, K'	[-]	constants in tool life equation
KB	[mm]	crater width
KB	[-]	reduction factor for width of the jaws
Kn	[-]	reduction factor for number of jaws
L...L3	[mm]	length of sections of the workpiece
l	[mm]	cutting length
MM	[mm]	spindle torque
n	[1/s]	number of revolutions
nj	[-]	number of jaws
p	[N/m ²]	hydraulic pressure
Rn	[mm]	roundness tolerance
s	[mm]	feed per revolution
T	[s]	tool life
ta	[s]	auxiliary time (set-up etc.)
tp	[s]	tool traverse and positioning time
ts	[s]	tool changing time
U	[m/N]	compliance
U1	[-]	constant in compliance equation
U2	[m/N]	compliance of the workpiece
Vo	[mm]	generalized wear criterion (KBo or VBo)
VB	[mm]	width of the flank wear land
v	[m/s]	cutting speed
Δx	[mm]	deflection of the workpiece
z	[mm]	co-ordinate along the centerline
u	[-]	friction co-efficient
ω	[rad/s]	angular velocity

References

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